Electroweak Bloch-Nordsieck violation at the TeV scale: "strong" weak interactions ?

Marcello Ciafaloni
Dipartimento di Fisica, Università di Firenze e
INFN - Sezione di Firenze, I-50125 Florence , Italy
E-mail: ciafaloni@fi.infn.it

Paolo Ciafaloni
INFN - Sezione di Lecce,
Via per Arnesano, I-73100 Lecce, Italy
E-mail: Paolo.Ciafaloni@le.infn.it

Denis Comelli
INFN - Sezione di Ferrara,
Via Paradiso 12, I-35131 Ferrara, Italy
E-mail: comelli@fe.infn.it

Abstract

Hard processes at the TeV scale exhibit enhanced (double log) EW corrections even for inclusive observables, leading to violation of the Bloch-Nordsieck theorem. This effect, previously related to the non abelian nature of free EW charges in the initial state (e^-e^+, e^-p, pp ...), is here investigated for fermion initiated hard processes and to all orders in EW couplings. We find that the effect is important, especially for lepton initiated processes, producing weak effects that in some cases compete in magnitude with the strong ones. We show that this (double log) BN violating effect has a universal energy dependence, related to the Sudakov form factor in the adjoint representation. The role of this form factor is to suppress cross section differences within a weak isospin doublet, so that at very large energy the cross sections for left-handed *electron*-positron and *neutrino*-positron scattering become equal. Finally, we briefly discuss the phenomenological relevance of our results for future colliders.

1 Introduction

Enhanced electroweak corrections at the TeV scale have been recently investigated by various authors [1]-[5] starting from the observation, made by two of us [1], that double and single logarithms of Sudakov type are present and sizeable in fixed angle fermion antifermion annihilation processes at NLC energies. These effects produce, namely, energy-growing corrections $\propto \alpha_W \log^2 \frac{s}{M^2}$, the weak scale $M \sim 90$ GeV providing a physical cutoff for infrared and collinear divergences.

In a recent paper [6] we have pointed out a different but related effect, which is peculiar of electroweak interactions. Due to the non abelian nature of electroweak charges in the initial state, the double logs persist at *inclusive* level, thus leading to violation of the Bloch-Nordsieck theorem [7], in the sense that the dependence on the IR cutoff M is not washed out when summing real and virtual corrections, as is usually the case. The peculiar aspect which makes such double logs observable is symmetry breaking itself, which generates the physical cutoff M on one hand, and allows the preparation of initial states as free abelian charges on the other*. In fact, the BN theorem is in principle violated in QCD also [8]-[12], but in such case confinement forces a color averaging in the initial state, which washes out the effect eventually. The "preconfinement" features, pointed out at various stages [11, 13] mean that free quark asymptotic states make no sense, even at perturbative level, because of form factors analogous to the ones discussed here. The situation is different of course in the EW case: the analogous of color averaging would mean for instance averaging over the cross sections for νe^+ and $e^- e^+$; this is meaningless from an experimental point of view.

In this paper we investigate the structure of double log EW corrections to all orders for light fermion initiated hard processes, and we characterize them by a universal energy dependence, related to the EW Sudakov form factor in the adjoint representation. The core of such analysis is provided by two lines of thought. One, explained in Sec.3, is based on the observation that only W contributions are BN violating, the Z and γ ones being canceled between virtual and real emission terms. The energy dependence is then universal because the W couples universally to left handed fermions and can be calculated to all orders by a simple Feynman diagram technique.

The other line of thought, explained in Sec.5 following the coherent state formalism of Sec.4, is based on the isospin structure of the overlap matrix, describing the squared matrix element. With a proper choice of indices, the singlet projection provides the isospin average (i.e. for instance $\sigma_{\nu e^+} + \sigma_{e^- e^+}$), and the vector projection the cross sections difference (i.e. $\sigma_{\nu e^+} - \sigma_{e^- e^+}$). Only the latter is suppressed by a Sudakov form factor, which refers to the adjoint representation, and is thus universal. The latter approach is made completely rigorous by the use of the general coherent state formalism [14, 15] introduced for cancellation theorems in Sec.2, and by the important observation that the photon scale plays no role for the (inclusive) BN violating terms. This fact follows from the explicit cancellation of Z and γ contributions found by the diagrammatic approach, so that the effect starts, and the symmetry is effectively restored at, the same W threshold M.

Applications to physical processes are analyzed in Sec.6 and discussed in Sec.7. We limit ourselves to lepton and/or quark large angle scattering as trigger process and we classify the results according to the initial state, which is provided by the accelerator. Important corrections are found, especially for lepton initiated processes, as in the case of $e\bar{e}$ (NLC) or ep and $\bar{e}p$. Roughly speaking, we find in this case a 50 percent reduction in hadron beams compared to lepton beams, mostly due to the hadrons acting as weak isospin mixtures in the initial state. Another important feature is the strong dependence of the BN violating effects on the polarization of the colliding beams, of particular relevance for NLCs [16].

2 Cancellation theorems and Bloch-Nordsieck violation

We consider the structure of soft interactions accompanying a hard SM process, of type

$$\{\alpha_1^I p_1^I, \alpha_2^I p_2^I\} \to \{\alpha_1^F p_1^F, \alpha_2^F p_2^F, \ldots, \alpha_n^F p_n^F\} \tag{1}$$

*For instance, an electron is prepared at low energy $\ll M$ as an abelian (QED) charge. As it is accelerated at energies $\gg M$, its charge acquires a fully nonabelian character.

where α, p denote isospin/color and momentum indices of the initial and final states, that we collectively denote by $\{\alpha_I p_I\}$ and $\{\alpha_F p_F\}$. The S matrix for such a process can be written as an operator in the soft Hilbert space $\mathcal{H}_S$, that collects the states which are almost degenerate with the hard ones, in the form

$$S = \mathcal{U}^F_{\alpha_F \beta_F}(a_s, a_s^\dagger) \quad S^H_{\beta_F \beta_I}(p_F, p_I) \quad \mathcal{U}^I_{\alpha_I \beta_I}(a_s, a_s^\dagger) \tag{2}$$

where $\mathcal{U}^F$ and $\mathcal{U}^I$ are operator functionals of the soft emission operators $a_s, a_s^\dagger$.

Eq. (2) is supposed to be of general validity [17, 15], because it rests essentially on the separation of long-time interactions (the initial and final ones described by the $\mathcal{U}$'s), and the short-time hard interaction, described by S^H. The real problem is to find the form of the $\mathcal{U}$'s , which is well known in QED [14], has been widely investigated in QCD [15], and is under debate in the electroweak case [4] -[5]. Their only general property is unitarity in the soft Hilbert space $\mathcal{H}_S$, i.e.

$$\mathcal{U}_{\alpha\beta}\mathcal{U}^\dagger_{\beta\alpha'} = \mathcal{U}^\dagger_{\alpha\beta}\mathcal{U}_{\beta\alpha'} = \delta_{\alpha\alpha'} \tag{3}$$

The key cancellation theorem satisfied by Eq. (2) is due to Lee, Nauenberg and Kinoshita [18], and states that soft singularities cancel upon summation over initial and final soft states which are degenerate with the hard ones:

$$\sum_{i\in\Delta(p_I)}^{f\in\Delta(p_F)} |\langle f|S|i\rangle|^2 = \mathrm{Tr}_{\mathcal{H}_S}(\mathcal{U}^{I\dagger}S^{H\dagger}\mathcal{U}^{F\dagger}\mathcal{U}^F S^H \mathcal{U}^I) = \mathrm{Tr}_{\alpha_I}(S^{H\dagger}(p_F, p_I) S^H(p_F, p_I)) \tag{4}$$

where $\Delta(p_I, p_F)$ denote the sets of such soft states, and we have used the unitarity property (3).

Although general, the KLN theorem is hardly of direct use, because it involves the sum over the initial degenerate set, which is not available experimentally. In the QED case, however, there is only an abelian charge index, so that $\mathcal{U}_I$ commutes with $S^{H\dagger}S^H$, and cancels out by sum over the final degenerate set only. This is the BN theorem: observables which are inclusive over soft final states are infrared safe.

If the theory is non abelian, like QCD or the electroweak one under consideration, the BN theorem is generally violated, because the initial state interaction is not canceled, i.e., by working in color space,

$$\sum_{f\in\Delta(p_F)} |\langle f|S|i\rangle|^2 = \ {}_S\langle 0|\mathcal{U}^{I\dagger}_{\alpha_I\beta_I'}(S^{H\dagger}S^H)_{\beta_I'\beta_I}\mathcal{U}^I_{\beta_I\alpha_I}|0\rangle_S = (S^{H\dagger}S^H)_{\alpha_I\alpha_I} + \Delta\sigma_{\alpha_I} \tag{5}$$

where the α_I indices are not summed over, and $\Delta\sigma_{\alpha_I}$ is, in general, nonvanishing and IR singular.
Fortunately, in QCD the BN cancellation is essentially recovered because of two features: (i) the need of initial color averaging, because hadrons are colorless, and (ii) the commutativity of the leading order coherent state operators ($\mathcal{U}^l$) for any given color indices [15]:

$$\mathcal{U}^l = \mathcal{U}^l(a_s - a_s^\dagger) \quad , \quad [\mathcal{U}^l_{\alpha\beta}, \mathcal{U}^l_{\alpha'\beta'}] = 0 \tag{6}$$

We obtain therefore

$$\sum_{color} \mathcal{U}^{l\dagger}_{\alpha_I\beta_I'}(S^{H\dagger}S^H)_{\beta_I'\beta_I}\mathcal{U}^l_{\beta_I\alpha_I} = \sum_{color}(S^{H\dagger}S^H)_{\beta_I'\beta_I}\mathcal{U}^l_{\beta_I\alpha_I}\mathcal{U}^{l\dagger}_{\alpha_I\beta_I'} = Tr_{color}S^{H\dagger}S^H \tag{7}$$

thus recovering an infrared safe result (for subleading features, see Refs. [10, 12, 11]).

In the electroweak case, in which M provides the physical infrared cutoff, there is no way out, because the initial state is prepared with a fixed non abelian charge. Therefore Eq. (5) applies, and double log corrections $\sim \alpha_W \log^2 \frac{s}{M^2}$ must affect any observable associated with a hard process, even the ones which are inclusive over final soft bosons. This fact is surprising, because one would expect such observables to depend only on energy and on running couplings, while the double logs represent an explicit M (infrared cutoff) dependence.

3 Lowest order calculation and picture of higher orders

In order to compute the uncanceled double logs, we first notice that, according to Eq. (5), only initial state interactions need to be taken into account. Here we give a diagrammatic account of the calculations, by considering EW corrections to the overlap matrix $O_H \equiv S_H^\dagger S_H$, in which soft bosonic lines are emitted and/or absorbed on initial lines only. In the following sections we shall give a more rigorous treatment, based on the coherent state approach that we have introduced previously.

We start treating the lowest order soft EW contributions to $\Delta\sigma \equiv \sigma - \sigma^H$ for a basic hard process involving a hard scale $E \gg M_W \sim M_Z \equiv M$, and two massless fermions. We consider here the case for two L initial fermions, both carrying nonabelian isospin (SU(2)) indices; a more general treatment is demanded to section 5. Since we only consider inclusive processes, a sum over degenerate final states is understood, and we drop the superscript indicating initial states: $\alpha_i^I \to \alpha_i$. In isospin space, the hard cross section structure is then defined by the so called hard overlap matrix, describing the squared matrix element: $\langle\beta_1\beta_2|S_H^\dagger S_H|\alpha_1\alpha_2\rangle \equiv (O_H)_{\beta_1\beta_2,\alpha_1\alpha_2}$ (see Fig. 1). While for cross sections we always have $\alpha_i = \beta_i$, we leave open the possibility that $\alpha_i \neq \beta_i$ and see O_H as an operator in isospin space with four indices.

Since pure QED corrections, at energies below M, cancel out automatically as noticed before, we limit ourselves to bosonic energies $M \ll w \ll E$, for which the gauge bosons γ, Z, W have all approximately the same momenta, with M acting as the only IR cutoff. This amounts eventually to setting the effective photon scale $\lambda = M$, and neglecting symmetry breaking effects at the inclusive double log level we are working. We would like to stress again the fact that this holds only for completely inclusive quantities, while for (partially) exclusive observables the presence of a new scale $\lambda \neq M$ is unavoidable and may lead to symmetry breaking effects, a topic which is currently under discussion [4, 5].

At lowest order, the above assumption is easily verified, because in the hard (Born) matrix element, since we work in a limit in which all kinematical invariants are much bigger than gauge bosons masses: $|s|, |t|, |u| \gg M^2$, the full gauge symmetry SU(3)⊗SU(2)⊗U(1) is restored. Boson emission and absorption is then described by the external (initial) line insertions of the eikonal currents

$$J_a^\mu = g[\frac{p_1^\mu}{kp_1}(t_1^a - t_1'^a) + \frac{p_2^\mu}{kp_2}(t_2^a - t_2'^a)] \qquad J_0^\mu = g'[\frac{p_1^\mu}{kp_1}(Y_1 - Y_1') + \frac{p_2^\mu}{kp_2}(Y_2 - Y_2')] \tag{8}$$

Here a=1,2,3 is the SU(2) index, we work in the unbroken basis $A_0 = c_W\gamma - s_W Z, A_3 = s_W A + c_W Z$, g and g' being the usual electroweak couplings with $\frac{s_W}{c_W} = \frac{g'}{g}$. The isospin operator $t_1(t_1')$ acts on the α_1 (β_1) index, and so on. The currents (8) are conserved ($k \cdot J = 0$) because of charge conservation at Born level:

$$t_1^a + t_2^a = t_1'^a + t_2'^a \quad , \quad Y_1 + Y_2 = Y_1' + Y_2' \tag{9}$$

Since both Y and t_3 are diagonal matrices, the A_0, A_3 (or γ, Z) contributions to the cross section *cancel out automatically* between virtual and real emission terms ($t_i^3 = t_i'^3, Y_i = Y_i'$). The W contributions are instead provided by:

$$g^2\frac{2p_1p_2}{(kp_1)(kp_2)}(\boldsymbol{t}_1 - \boldsymbol{t}_1') \cdot (\boldsymbol{t}_2 - \boldsymbol{t}_2') \qquad (\boldsymbol{t}_1 \cdot \boldsymbol{t}_2 \equiv \sum_a t_1^a t_2^a) \tag{10}$$

where we have kept, for notational convenience, the vanishing A_3 contribution, and the charge factor can be replaced, because of the conservation (9), by

$$(\boldsymbol{t}_1 - \boldsymbol{t}_1') \cdot (\boldsymbol{t}_2 - \boldsymbol{t}_2') = -(\boldsymbol{t}_1 - \boldsymbol{t}_1')^2 = 2\boldsymbol{t}_1 \cdot \boldsymbol{t}_1' - \boldsymbol{t}_1^2 - \boldsymbol{t}_1'^2 \tag{11}$$

The latter expression provides the charge computation in the axial gauge, because in this gauge the W emission and absorption takes place on the same leg, for both virtual ($-2\boldsymbol{t}_1^2$) and real emission ($2\boldsymbol{t}_1 \cdot \boldsymbol{t}_1'$) contributions. We can see from Fig. 1 the reason for noncancellation between virtual and real one loop corrections: the crucial point is that while γ, Z emission does not change the initial state, W emission does. Then, in the W case virtual corrections (Fig. 1a) are proportional to $\sigma_{e\bar{e}}$, while real corrections (Fig. 1b) are of opposite sign but proportional to $\sigma_{\nu\bar{e}} \neq \sigma_{e\bar{e}}$, giving rise to a non complete cancellation.

In order to simplify our considerations, let us note that, because of isospin conservation, we must have (see (44))

$$\sigma_{\nu\bar{\nu}} = \sigma_{e\bar{e}} \quad , \quad \sigma_{\bar{\nu}e} = \sigma_{\nu\bar{e}} \tag{12}$$

We can therefore limit ourselves to the cross sections $\sigma_\alpha \equiv \sigma_{\alpha\bar{e}}$ where $\alpha = \nu, e$ is a single isospin index. Since real W emission changes the isospin index, while virtual corrections don't, we can summarize the first order calculations based on (8) and (10) in the form

$$\Delta\sigma^1_\alpha = L_W(s)[-\delta_{\alpha\beta} + (\tau_1)_{\alpha\beta}]\sigma^H_\beta \tag{13}$$

where τ_1 is the customary Pauli matrix, the superscript 1 denotes the order of correction, and

$$L_W(s) = \frac{g^2}{2}\int_M^E \frac{d^3\boldsymbol{k}}{2w_k(2\pi)^3}\frac{2p_1p_2}{(kp_1)(kp_2)} = \frac{\alpha_W}{4\pi}\log^2\frac{E^2}{M^2} \qquad (\alpha_W = \frac{g^2}{4\pi}) \tag{14}$$

is the eikonal radiation factor for W exchange.

From Eq. (13) we get in particular:

$$\Delta\sigma^1_e = L_W(s)(\sigma^H_\nu - \sigma^H_e) = -\Delta\sigma^1_\nu \tag{15}$$

which was the main result of [6], leading also to $\Delta\sigma^1_e + \Delta\sigma^1_\nu = 0$, i.e., to cancellation upon isospin averaging.

The insertion mechanism just explained can be iterated to higher orders in the strong ordering region $M \ll w_1 \ll w_2 ..... \ll E$, by noticing that the current (8), after cancellation of the γ, Z contributions takes up only a W part with only one cutoff (M), which cannot induce symmetry breaking. Since the action of real W emission is simply that of interchanging ν and e indices, we end up with a two channel problem where the energy evolution hamiltonian - derived from eq. (13) - commutes at different energies. The outcome is a simple exponentiation of the first order result:

$$\sigma_\alpha(s) = \sigma^H_\alpha + \sum_{n=0}^{\infty}\Delta\sigma^n_\alpha = \{\exp L_W(s)(\tau_1 - 1)\}_{\alpha\beta}\sigma^H_\beta \quad , \tag{16}$$

or, by simple algebra,

$$\sigma_{e,\nu} = \frac{\sigma^H_\nu + \sigma^H_e}{2} \mp \frac{\sigma^H_\nu - \sigma^H_e}{2}e^{-2L_W(s)} \tag{17}$$

This means that on average the double logs cancel out ($\sigma_{e\bar{e}} + \sigma_{\nu\bar{e}} = \sigma^H_{e\bar{e}} + \sigma^H_{\nu\bar{e}}$), while the ν-beam and e-beam difference $\sigma_{\nu\bar{e}} - \sigma_{e\bar{e}}$ decreases exponentially with the universal exponent $2L_W(s)$, and vanishes eventually at infinite energy.

Apart from the important phenomenological implications of Sec.6, the above results have a nice theoretical interpretation in a non abelian framework, which will be illustrated in the following sections. First, the exponent $2L_W(s)$ can be related to the form factor in the adjoint representation, which is in fact a typically non abelian quantity; because of gauge invariance the exponent is also universal, i.e. the same for any fermion doublet in the initial state. A similar relationship was noticed for QCD in [9, 11]. Furthermore, the asymptotic equality of neutrino and electron beam cross sections is related to the idea that in the unbroken limit ($M \to 0$), nonabelian charges are actually not observable as asymptotic states, as already noticed in QCD in connection with preconfinement [13] and factorization violating contributions [11]. In fact, if there were no symmetry breaking (infinite $2L_W(s)$), Eq. (17) would imply that any coherent superposition of electron and neutrino states is projected by the nonabelian interaction into an incoherent mixture with equal weights.

In the following sections we shall further elaborate on the heuristic result of Eq. (17), by providing a more general classification, and a proof based on the coherent state operator formalism adopted for the cancellation theorems of Sec.2.

4 Leading coherent state operators

The evaluation of BN violating terms - just pictured in the diagrammatic approach - rests on the separation of initial and final state soft interactions, that can be performed on the basis of the time evolution in a given reference frame, e.g. the c.m. frame of the initial state. This method, first applied by Faddeev and Kulish [14] to QED, has been extended to (unbroken) non abelian theories in Refs. [15], by constructing the large time (asymptotic) Hamiltonian as an infinite series of progressively subleading contributions.

Here we limit ourselves to describe such analysis at the leading (double log) level. This does not mean that subleading contributions are unimportant; it just means that the state of the art in a broken theory is not sophisticated enough, at present, to allow confidence in all-order subleading evaluations, even for the inclusive quantities that we investigate here. In particular, collinear factorization theorems should be carefully revised, partly because of the very presence of the uncanceled double logs that we are computing.

4.1 Quantum Electrodynamics

According to Eq. (2), the process is separated into a hard scattering matrix S_H, involving the scale $E \gg \lambda$ (λ is the IR cutoff), and in a soft interaction, described by $\mathcal{U}^I$ and $\mathcal{U}^F$, involving photon frequencies w such that $E \gg w \gg \lambda$. In describing the soft interaction, the hard (initial and final) fermions are treated as external currents. Thus, the interaction Hamiltonian describing the evolution in the soft Hilbert space $\mathcal{H}_s$ is simply:

$$H_s(t) = \sum_i e_i \int_\lambda^E d[k]\hat{p}_i^\mu \left(A_\mu(\boldsymbol{k})e^{-i(\hat{p}_i k)t} + h.c.\right) \equiv \int d\nu (h_+(\nu)e^{-i\nu t} + h_-(\nu)e^{i\nu t}) \tag{18}$$

$$h_+(\nu) = \sum_i e_i \int_\lambda^E d[k]\hat{p}_i^\mu A_\mu(\boldsymbol{k})\delta(\nu - \hat{p}_i k) \tag{19}$$

where $d[k] = \frac{d^3\boldsymbol{k}}{2w_k(2\pi)^3}$, $E_i\hat{p}_i^\mu = p_i^\mu$ ($i = 1, ...n_I$) denote the momenta of the hard particles in the initial (or final) state, $k^\mu = (w_k, \boldsymbol{k})$ is the photon momentum, and $A_\mu(\boldsymbol{k})$ ($A_\mu^\dagger(\boldsymbol{k})$) denote the photon annihilation (creation) operators. The "energy transfer" variable $\nu = \hat{p}_i k$ is therefore conjugated to the time variable t in this approach, and represents physically the quantity of energy transferred in an elementary vertex.

The key feature of Eq. (18) is the (universal) eikonal coupling of charged hard particles to photons, proportional to their velocities $\hat{p}^\mu$. At amplitude level, this implies the insertion formulas with the eikonal current

$$J^\mu(k) = \sum_i e_i \frac{p_i^\mu}{kp_i} \quad , \tag{20}$$

whose non abelian counterpart has been given in Eq. (8).

The in (out) coherent state operators occurring in Eq. (2) are obtained from the soft Hamiltonian (18) by computing the time-ordered evolution operator before (after) the hard scattering. In the QED case, the calculation is simplified by the fact that H_s is linear in the A_μs, and has therefore c-number commutators at non equal times. For instance we obtain, by standard methods [14], the initial state operator

$$\mathcal{U}_I = U(0, -\infty) = e^{i\phi_C} \exp\left[\int_\lambda^E \frac{d\nu}{\nu}(h_+(\nu) - h_-(\nu))\right] \qquad \phi_C = 2\pi \int_\lambda^E \frac{d\nu}{\nu}[h_+(\nu), h_-(\nu)] \tag{21}$$

Let us now consider for simplicity the case of an initial state of two particles 1 and 2 with opposite charges[†] and relative velocity v_{12}. By using the explicit form of the h's we obtain:

$$\mathcal{U}_I = U_1U_2 = \exp \int_\lambda^E d[k]J_{12}^\mu(k)(A_\mu(\boldsymbol{k}) - A_\mu^\dagger(\boldsymbol{k})) = :\mathcal{U}_I: \exp[\int_\lambda^E d[k][J_{12}(k)]^2 \tag{22}$$

[†]For general charge configurations, a gauge invariant picture requires considering initial and final state operators all together

$$\phi_C = \frac{\alpha}{2v_{12}} \qquad J_{12}^\mu = e(\frac{p_1^\mu}{kp_1} - \frac{p_2^\mu}{kp_2})$$

Since the Coulomb phase ϕ_C doesn't give physical effects for the processes considered here, we shall drop it from now on.

Note that the operator $\mathcal{U}_I$ is factorized in the particle indices $i = 1, 2$ and is a functional of the combination $A_\mu(\boldsymbol{k}) - A_\mu^\dagger(\boldsymbol{k}) \equiv i\Pi_\mu(\boldsymbol{k})$ only. For this reason the currents J_i^μ can be freely added in the exponent, and unitarity is trivial.

A less trivial, but straightforward step is the normal ordering quoted in Eq. (22), which emphasizes the Sudakov form factor. In the massless limit we obtain

$$F_{12} \equiv \langle 0|U(1)U(2)|0\rangle = \exp\left[-e^2 \int_\lambda^E d[k] \frac{2p_1 p_2}{(kp_1)(kp_2)}\right] \approx \exp[-\frac{\alpha}{4\pi}\log^2\frac{E^2}{\lambda^2}] \tag{23}$$

From the same normal ordering formula one can obtain the (Poissonian) properties of soft photon radiation, the energy resolution form factors, and so on.

4.2 Non abelian coherent states

The asymptotic Hamiltonian has been constructed [15] in the (unbroken) SU(N) gauge theory also. The difficulty, in such case, is that soft bosons can be either primaries (i.e. emitted directly by the fast incoming particles) or secondaries (i.e. emitted by the primary ones which have higher energy). If strong ordering in energies is assumed, the soft Hamiltonian has nevertheless a simple form, as follows:

$$H_s(t) = g\int_\lambda^E d[k]\left[\left(\sum_i t_i^a \hat{p}_i^\mu e^{-i(\hat{p}_i k)t} + \int_{w_k}^E d[k']\rho^a(\boldsymbol{k}')\hat{k}'^\mu e^{-i(\hat{k}'k)t}\right) A_{\mu a}(\boldsymbol{k}) + \text{ h.c.}\right] \tag{24}$$

$$\rho^a(\boldsymbol{k}) = -A^\dagger_{\mu b}(\boldsymbol{k})(T^a)^{bc}A^\mu_c(\boldsymbol{k}) \quad , \quad (T^a)^{bc} = if^{bac}$$

In Eq. (24), the second term in round brackets represents the secondary emission by primary bosons with energies $w_{k'} \gg w_k$, which occurs again through an eikonal vertex $\hat{k}'^\mu \equiv \frac{k'^\mu}{w_{k'}}$. Because of the assumption of energy ordering, we are limiting ourselves to the leading coherent state operators, and we refer to [15] for the subleading contributions.

At this level of accuracy, we can compute the evolution operator by assuming strong ordering in the energy transfers also (for details, see [15]). By indicating with P_ω the energy ordering operator (smaller energies act first), and by introducing the field $\Pi_{\mu a}(k) \equiv -i(A_{\mu a}(k) - A^\dagger_{\mu a}(k))$, we obtain

$$\begin{aligned} U_s(0,-\infty) &= P_\nu \exp\left[\int_\lambda^E \frac{d\nu}{\nu}\left(h_+(\nu) - h_-(\nu)\right)\right] \equiv V_s^E U_I^l \\ &= P_\omega \exp\left[ig\int_\lambda^E d[k]\left(\sum_i t_i^a \frac{p_i^\mu}{kp_i} + \int_w^E d[k']\rho^a(\boldsymbol{k}')\frac{k'^\mu}{kk'}\right)\Pi_{\mu a}(\boldsymbol{k})\right] \quad ; \\ V_s^E &\equiv P_\omega \exp\left[ig\int_\lambda^E d[k]\int_{w_k}^E d[k']\rho^a(\boldsymbol{k}')\frac{k'^\mu}{kk'}\Pi_{\mu a}(\boldsymbol{k})\right] \end{aligned} \tag{25}$$

Here we face the problem of explicitating the recursive emission of secondaries. We treat the full path-ordered exponential in Eq. (25) as a two-potential problem, one of which produces the purely soft operator V_s, which carries no isospin/color indices, and the other one is computed in the interaction representation of the first, and provides the properly called coherent state operator $\mathcal{U}_I^l$, carrying the isospin/color indices of the

incoming particles. The result of this procedure is

$$\mathcal{U}_I^l \equiv V_s^\dagger U_s(0,-\infty) = P_\omega \exp\left[ig\int_\lambda^E d[k]\sum_I t_i^a \frac{p_i^\mu}{kp_i}\Pi_{\mu a}^{w_k}(\boldsymbol{k})\right] \tag{26}$$

$$\mathcal{U}_F^l \equiv U_s^\dagger(0,-\infty) V_s = \bar{P}_\omega \exp\left[-ig\int_\lambda^E d[k]\sum_F t_f^a \frac{p_f^\mu}{kp_f}\Pi_{\mu a}^{w_k}(\boldsymbol{k})\right] \tag{27}$$

where $\bar{P}_\omega$ is the energy anti-ordering operator (bigger energies act first) and where the "dressed" field $\Pi_{\mu a}^w$ can be made more explicit as follows

$$\Pi_{\mu a}^w \equiv V_s^{w\dagger}\Pi_{\mu a}V_s^w = (U_A^\omega)_{ab}(\hat{\boldsymbol{k}})\Pi_{\mu b}(\boldsymbol{k}) \tag{28}$$

where $(U_A^\omega)_{ab}(\hat{\boldsymbol{k}})$ denotes the coherent state operator in the *adjoint* representation, for a boson $w\hat{k}^\mu$, having the matrix properties (recall $(T_a)_{bc} = if_{bac}$)

$$U_A^* = U_A \qquad U_A^T = U_A^\dagger = U_A^{-1} \tag{29}$$

Eq. (28) was proved in Ref [15], by showing that the l.h.s. and the r.h.s. satisfy the same evolution equation in the w variable.

Eqs. (26-29) show that non abelian coherent states operators are defined in a quite nonlinear manner. Eq. (26) exponentiates up to energy E the dressed fields Π^ω, which in turn are expanded in terms of a coherent state at a lower energy $w < E$ by Eq. (28), which in turn etc...... In other words non abelian soft radiation is described by a "coherent state of lower energy coherent states" whose structure is obtained recursively by a branching process [15], by now incorporated in some QCD event generators [19].

For our purpose here, the important point is that we are now able to explicitate the soft part of the process (1) as follows

$$S = U_s^\dagger(0,+\infty) S_H U_s(0,-\infty) = \mathcal{U}_F^l S_H \mathcal{U}_I^l \tag{30}$$

Here the operator V_s defined in Eq. (25), being purely soft, commutes with S_H and drops out by unitarity, and $\mathcal{U}_F^l$,$\mathcal{U}_I^l$ are provided by Eqs. (26) and (28) at leading level.

The leading operator $\mathcal{U}^l$ - despite its nonlinearity - is still a functional of the fields $\Pi_{\mu a}(\boldsymbol{k})$ only, and involves therefore only *commuting* quantities in the Fock space. It follows, therefore, that commutativity for any given color indices holds:

$$[\mathcal{U}_{\beta\alpha}^l, \mathcal{U}_{\beta'\alpha'}^l] = 0 \tag{31}$$

as already used in Sec.2, and that factorization with respect to the hard particles holds also

$$\mathcal{U}_{\beta\alpha}^l(1,\ldots,n) = \mathcal{U}_{\beta_1\alpha_1}^l(1)\,\mathcal{U}_{\beta_2\alpha_2}^l(2)\,\ldots\,\mathcal{U}_{\beta_n\alpha_n}^l(n) \qquad . \tag{32}$$

Finally, one should note that the coherent state in the adjoint representation regulates the evolution equation of t^a matrices:

$$U^{E\dagger}(\boldsymbol{p}) t_a U^E(\boldsymbol{p}) = (U_A^E)_{ab}(\boldsymbol{p}) t_b \tag{33}$$

for an arbitrary isospin/color representation of the particle $\boldsymbol{p}$. In fact, the coherent state operators satisfy the Schrödinger-like equation

$$\frac{\partial}{\partial E}U^E(\boldsymbol{p}) = t_b\Delta_b^E U^E(\boldsymbol{p}) \qquad \Delta_b^E = ig\int d[k]\delta(w_k - E)\frac{p^\mu}{kp}\Pi_{\mu b}^E(\boldsymbol{k}) \tag{34}$$

Therefore, by combining the U and $U^\dagger$ equations, the l.h.s. of Eq. (33) satisfies the equation

$$\frac{\partial}{\partial E}\mathrm{Tr}(U^{\dagger E} t_a U^E t_d) = if_{abc}\Delta_b^E \mathrm{Tr}(U^{\dagger E} t_c U^E t_d) \tag{35}$$

which is the same as the one satisfied by the r.h.s., by direct use of (34) in the adjoint representation.

4.3 Form factor exponentiation

The Sudakov (singlet) form factor can be defined similarly to Eq. (23) (with opposite initial charges), for any given color representation of particles 1 and 2:

$$\langle 0| \left(U^{E\dagger}(2)U^{E}(1)\right)_{\alpha\beta} |0\rangle = \delta_{\alpha\beta}F_{12}(E,\lambda) \tag{36}$$

However, due to the nonlinearity of Eqs. (26) and (28) the normal ordering is no longer straightforward, and was worked out by an evolution equation method in Ref. [15].

Since the energy ordered exponential satisfies the Schrödinger-like equation (34), one first computes the energy derivative of (33), using also definition (28), as follows ($w_K = E$)

$$\mathrm{Tr}(1)\frac{\partial}{E\partial E}F_{12} = \langle 0|\mathrm{Tr}\left(U^{E\dagger}(2)\frac{gd\Omega_K}{2(2\pi)^3}J^{\mu}_{12}t_a(U_A)^E_{ab}(\hat{K})(A_{\mu b}(\boldsymbol{K}) - A^{\dagger}_{\mu b}(\boldsymbol{K}))U^E(1)\right)|0\rangle \tag{37}$$

where $J^{\mu}_{12} = \frac{p_1^{\mu}}{Kp_1} - \frac{p_2^{\mu}}{Kp_2}$. Then, one commutes $A_{\mu b}(K)$ to the right and $A^{\dagger}$ to the left. Since $[A_{\mu b}(\boldsymbol{K}), A^{w_k \dagger}_{\nu a}(\boldsymbol{k})] = 0$ for $w_k < E$ this procedure singles out the upper frequency in U^E and yields

$$[A_{\mu b}(\boldsymbol{K}), U^E(1)] = g(U_A)^E_{cb}\frac{p_1^{\mu}}{Kp_1}t_c U^E(1) \tag{38}$$

with a similar relation involving $A^{\dagger}$ and p_2^{μ}. Finally, by using the unitarity relation

$$\sum_b (U^E_A)_{ab}(i)(U^E_A)_{cb}(i) = \delta_{ac} \qquad (i = 1, 2) \tag{39}$$

and $\boldsymbol{t}_i^2 = C_F$ (or C_A, depending on the particle's representation) we obtain

$$\frac{\partial}{E\partial E}F_{12}(E,\lambda) = \frac{g^2 C_{F(A)}}{2(2\pi)^3}\left(\int_{w_K=E} d\Omega_K (J_{12}(K))^2\right)F_{12}(E,\lambda) \tag{40}$$

which is the evolution equation for the form factor we were looking for. In the double log approximation, (40) yields

$$F_{12}(E,\lambda) = \exp[-\frac{g^2 C_{F(A)}}{16\pi^2}\log^2\frac{E^2}{\lambda^2}] \tag{41}$$

as expected. Note that, in the $SU(2)$ isospin case, $C_A = 2$ and the exponent in Eq. (41) becomes just $2L_W(s)$.

The essential point of this derivation rests on the unitarity relation (39), which means the cancellation of all correlation effects due to the nonabelian structure. This leads to Eq. (40), which could be naively derived by the "external line insertion rule" for virtual corrections.

5 Bloch Nordsieck violation to all orders

5.1 Isospin structure of the hard overlap matrix

In this section we discuss the general structure of the hard overlap matrix, describing the squared matrix element: $O_H \equiv S^{\dagger}_H S_H$. We consider the case of a generic process with two partons in the initial state and we work in a limit in which all kinematical invariants are much bigger than gauge bosons masses: $|s|, |t|, |u| \gg M^2$. Then, the full gauge symmetry SU(3)⊗SU(2)⊗U(1) is restored, and the structure of the overlap matrix in isospin space is fixed by the SU(2) symmetry.

Left particles carry nonabelian $SU(2)$ charges while right particles don't, so we need to consider three cases:

- when both initial particles are righthanded, and therefore do not carry any nonabelian weak charge nor any isospin index, the overlap matrix is simply a number O^H in isospin space (still depending of course on the quantum numbers of the involved particles).

- Next possibility is that one particle is left polarized and the other one is right polarized. In this case the hard overlap matrix carries two (left) isospin indices $O^H_{\beta\alpha}$.
- The case of two left initial fermions, is of course the most complicated one; the hard overlap matrix carries in this case four isospin indices $O^H_{\beta_1\beta_2,\alpha_1\alpha_2}$ (see Fig 5).

While for cross sections we always have $\alpha_i = \beta_i$, we leave open the possibility that $\alpha_i \neq \beta_i$, and see O^H as an operator in isospin space. The form of the overlap matrix is severely restricted by the requirement of SU(2) symmetry. In the three cases discussed above, we have:

$$RR : O^H = A_0 \qquad RL, LR : O^H_{\beta,\alpha} = B_0\delta_{\beta\alpha} \qquad LL : O^H_{\beta_1\beta_2,\alpha_1\alpha_2} = C_0\delta_{\beta_1\alpha_1}\delta_{\beta_2\alpha_2} + C_1 4t^a_{\beta_2\alpha_2}t^a_{\beta_1\alpha_1} \tag{42}$$

The last expression, where the indices α_1, β_1 (and α_2, β_2) are grouped together, corresponds to a t-channel decomposition in singlet and vector components (see Fig. 6). The case where the initial particle on leg 2 is an antiparticle, thus belonging to the conjugate representation $t^* = t^T$, is correspondingly:

$$\bar{O}^H_{\beta_1\beta_2,\alpha_1\alpha_2} = \bar{C}_0\delta_{\beta_1\alpha_1}\delta_{\beta_2\alpha_2} + \bar{C}_1\, 4t^a_{\alpha_2\beta_2}t^a_{\beta_1\alpha_1} \qquad \text{(part - antipart)} \tag{43}$$

Let us consider as an example a generic hard cross section involving a left e^- and a left e^+ (which we indicate with e and $\bar{e}$), and $\nu, \bar{\nu}$[‡]. We have:

$$\sigma^H_{e\bar{e}} = \sigma^H_{\nu\bar{\nu}} \propto \bar{O}^H_{11,11} = \bar{O}^H_{22,22} = \bar{C}_0 + \bar{C}_1 \tag{44a}$$

$$\sigma^H_{e\bar{\nu}} = \sigma^H_{\nu\bar{e}} \propto \bar{O}^H_{12,12} = \bar{O}^H_{21,21} = \bar{C}_0 - \bar{C}_1 \tag{44b}$$

5.2 Resummed energy dependence from coherent states

We now proceed to "dress" the hard matrix element with soft interactions. In the case of right initial particles, since weak interactions become purely abelian in this case, it should be clear from the above discussion that no BN-violating effect is present. If one particle is L and the other one is R, the dressing by soft interactions is described by (5):

$$O^H_{\alpha\beta} \stackrel{dress}{\to} O_{\alpha\beta} = {}_S\langle 0|\mathcal{U}^\dagger_{\alpha\alpha'} O^H_{\alpha'\beta'} \mathcal{U}_{\beta'\beta}|0\rangle_S \tag{45}$$

But then, since by SU(2) symmetry $O^H_{\alpha\beta} = B_0\ \delta_{\alpha\beta}$, and because of the unitarity property (3), also in this case no BN violating effect is present and the dressed overlap matrix is equal to the hard one. In the remaining of this section we discuss the interesting case of two left initial fermions. As we have seen, the dressing in this case is described by a coherent state operator $\mathcal{U}^I$ such that (see Fig. 5):

$$O^H_{\beta_1\beta_2,\alpha_1\alpha_2} \stackrel{dress}{\to} O_{\beta_1\beta_2,\alpha_1\alpha_2} = {}_S\langle 0|\mathcal{U}^{I\dagger}_{\beta_1\beta_2,\beta'_1,\beta'_2}(O_H)_{\beta'_1\beta'_2,\alpha'_1\alpha'_2}\mathcal{U}^I_{\alpha'_1\alpha'_2,\alpha_1,\alpha_2}|0\rangle_S \tag{46}$$

where $|0\rangle_S$ is the soft vacuum. At the leading log level, $\mathcal{U}^{I\dagger}$ is factorized (Sec. 4.2) into two leg operators:

$$\mathcal{U}^I_{\alpha'_1\alpha'_2,\alpha_1,\alpha_2} = U^{(1)}_{\alpha'_1\alpha_1}U^{(2)\dagger}_{\alpha_2\alpha'_2} \quad \text{part-antipart} \qquad \mathcal{U}^I_{\alpha'_1\alpha'_2,\alpha_1,\alpha_2} = U^{(1)}_{\alpha'_1\alpha_1}U^{(2)}_{\alpha'_2\alpha_2} \quad \text{part-part} \tag{47}$$

where we take into account that antiparticles live in the conjugate representation, so that in the particle-antiparticle case we have $U^{(2)} \to U^{(2)*}$. Putting together (43,46,47) we obtain, for the part - antipart case, the dressed overlap operator:

$$\begin{aligned}\bar{O}_{\beta_1\beta_2,\alpha_1\alpha_2} &\equiv \bar{C}_0(s)\delta_{\beta_1\alpha_1}\delta_{\beta_2\alpha_2} + \bar{C}_1(s)\ 4t^a_{\beta_1\alpha_1}t^a_{\alpha_2\beta_2} \\ &= \bar{C}_0\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_2} + 4\bar{C}_1\ {}_S\langle 0|(U^{(1)\dagger}\, t^a\, U^{(1)})_{\beta_1\alpha_1}(U^{(2)\dagger}\, t^a\, U^{(2)})_{\alpha_2\beta_2}|0\rangle_S\end{aligned} \tag{48}$$

‡In our notation a particle and its antiparticle share the same isospin index: 1 for $\nu, \bar{\nu}$ and 2 for $e, \bar{e}$

By using twice Eq. (33) that relates the coherent states in the fundamental representation with the one in the adjoint representation, we obtain:

$$
{}_S\langle 0|(U^{(1)\dagger}t^a U^{(1)})_{\beta_1\alpha_1}(U^{(2)\dagger}t^a\, U^{(2)})_{\alpha_2\beta_2}|0\rangle_S = {}_S\langle 0|\left(U_A^{(2)\dagger}U_A^{(1)}\right)_{ab}|0\rangle_S t^a_{\beta_1\alpha_1}t^b_{\alpha_2\beta_2} = F_A(s,M^2)t^a_{\beta_1\alpha_1}t^a_{\alpha_2\beta_2} \tag{49}
$$

where the definition (36) of the form factor has been used, so that we obtain:

$$
\bar{C}_0(s) = \bar{C}_0 \qquad\qquad \bar{C}_1(s) = \bar{C}_1 e^{-C_A L_W(s)} = \bar{C}_1 e^{-2L_W(s)} \tag{50}
$$

From these expression we obtain the final results for the dressed cross sections (see Fig. 4):

$$
\sigma_{11} = \sigma_{22} = \bar{C}_0(s) + \bar{C}_1(s) = \bar{C}_0 + \bar{C}_1 e^{-2L_W(s)} = \frac{(\sigma_{11}+\sigma_{12})^H}{2} + \frac{(\sigma_{11}-\sigma_{12})^H}{2}\, e^{-2L_W(s)} \tag{51a}
$$

$$
\sigma_{12} = \sigma_{21} = \bar{C}_0(s) - \bar{C}_1(s) = \bar{C}_0 - \bar{C}_1 e^{-2L_W(s)} = \frac{(\sigma_{11}+\sigma_{12})^H}{2} - \frac{(\sigma_{11}-\sigma_{12})^H}{2}\, e^{-2L_W(s)} \tag{51b}
$$

which reproduce Eq.(17), and the relative effects in double log approximation:

$$
\left(\frac{\Delta\sigma}{\sigma}\right)_{11} \equiv \frac{\sigma_{11}-\sigma_{11}^H}{\sigma_{11}^H} = \left(\frac{\sigma_{11}^H - \sigma_{12}^H}{\sigma_{11}^H}\right)\left(\frac{1-e^{-2L_W(s)}}{2}\right) \tag{52a}
$$

$$
\left(\frac{\Delta\sigma}{\sigma}\right)_{12} \equiv \frac{\sigma_{12}-\sigma_{12}^H}{\sigma_{12}^H} = \left(\frac{\sigma_{12}^H - \sigma_{11}^H}{\sigma_{12}^H}\right)\left(\frac{1-e^{-2L_W(s)}}{2}\right) \tag{52b}
$$

An analogous treatment for the part - part case allows one to conclude that Eqns. (51,52) hold also for this case with the obvious replacement $\bar{C}_i \to C_i$. These final results are therefore completely general: σ_{11} stands for any cross section with two incoming particles (or one particle and one antiparticle) with isospin index 1, which means for instance $\sigma_{\nu\bar{\nu}}, \sigma_{uu}, \sigma_{u\bar{u}}$ and so on. While in all these cases the expressions for the coefficients C_0 and C_1 (or $\bar{C}_0$ and $\bar{C}_1$) that describe the hard cross section are of course different in general, the expression for the dressed cross sections is always the same and is described by Eqns. (51).

Notice the appearance, as anticipated, of the adjoint Casimir $C_A = 2$ in (51). This means that the energy dependence of the effect we are discussing is *universal*, i.e. the same for any fermion doublet in the initial state. Note however that the relative effect does depend on the structure of the hard cross sections, as one can see from (52). We will discuss several cases of phenomenological interest more in detail in Sec. 6.

6 Applications to simple processes

We consider inclusive observables associated to a large angle hard scattering process ($|s| \sim |t| \sim |u| \gg M^2$) involving massless fermions and antifermions in the initial and final states. As we have seen, a BN violating effect is present only if there are *two* nonabelian charges in the initial state. This means that big noncancellations are present only with two left fermions in the initial state, while the effect is absent in the RR and RL cases. In turn, this implies a strong dependence on the physical polarization of the initial beams: a maximal effect if the beams are both polarized L, no effect in all other cases, and somewhere in between for unpolarized beams. This is particularly important since one of NLCs features is the possibility of having highly polarized beams [16].

We discuss in this section some cases that we think are/will be phenomenologically relevant for NLCs (Sec. 6.1), and for electron-hadron (Sec. 6.2) and hadron-hadron (Sec. 6.3) colliders. Given the master formulas (51), and since the energy dependence is universal as already noticed, it is clear that only the (tree level) hard cross sections need to be discussed. We expect large effects when LL contributions dominate the hard cross section, and/or when there is a big difference between σ_{12} and σ_{11} (see (52)). The latter is the case, for instance, in pure $q\bar{q}$ s-channel annihilation where σ_{11} is of order α_S^2 while σ_{12} is flavor changing and is thus electroweak (Sec. 6.3). Similarly, in $e^+e^- \to$ hadrons (Sec. 6.1), the effect is pretty large because this process is dominated by L components.

6.1 $l\bar{l} \to q\bar{q}$ (s-channel) annihilation

This kind of process is typically relevant for NLC. The Born (hard) amplitude is simply described by an s-channel annihilation involving only weak interaction and has the form (a=1,2,3):

$$\mathcal{M}_{\beta_1\beta_2,\alpha_1\alpha_2} \sim g'^2 yY\delta_{\beta_1\beta_2}\delta_{\alpha_1\alpha_2} + g^2 t^a_{\beta_1\beta_2} t^a_{\alpha_1\alpha_2} \tag{53}$$

where y (Y) denote the initial lepton (final quark) hypercharges. Expression (53) is the s-channel analogue of the singlet-vector t-channel decomposition (42). By squaring and summing over final states, we obtain the overlap matrix t-channel components, as defined in (42), for the case of (initial) L fermions:

$$C_1^L = -\frac{1}{16}g^4 + \frac{1}{2}g'^4 y_L^2 \sum Y_L^2 \qquad C_0^L = \frac{3}{16}g^4 + \frac{1}{2}g'^4 y_L^2 \sum Y_L^2 \tag{54a}$$

$$C_0^R = \frac{1}{2}g'^4 y_L^2 \sum Y_R^2 \qquad C_1^R = \frac{1}{2}g'^4 y_L^2 \sum Y_R^2 \tag{54b}$$

where $C_i^{L(R)}$ refers to final L(R) quarks. For the case of initial R leptons we obtain:

$$A_0^R = \delta_{ij}\, g'^4 y_R^2 \sum Y_R^2 \qquad A_0^L = \delta_{ij}\, g'^4 y_R^2 \sum Y_L^2 \tag{54c}$$

where the factor δ_{ij} in (54c) takes into account that the contribution $\propto g'^4$ is present only for the case of an initial particle and its own antiparticle. The sums are over final states hypercharges, namely $\sum Y_R^2 = \frac{4}{9} + \frac{1}{9}$ and $\sum Y_L^2 = \frac{1}{36} + \frac{1}{36}$ for quarks. The unpolarized cross section is then found by putting in the usual phase space factor, and by using the resummed energy evolution in Eq. (50). We find:

$$\frac{d\sigma_{ij}}{d\cos\theta} = \frac{N_c N_f}{128\pi s}\left[(A_0^R + C_0^L \pm C_1^L e^{-2L_W(s)})(1+\cos\theta)^2 + (A_0^L + C_0^R \pm C_1^R e^{-2L_W(s)})(1-\cos\theta)^2\right] \tag{55}$$

where N_f is the number of quark families, N_c is the number of colors and the + (-) sign refers to σ_{11} (σ_{12}).

The terms proportional to g'^4 in Eq. (55) are pretty small, being suppressed by a $\frac{g'^4}{g^4} = t_W^4$ factor. In the limit $g' \to 0$ only L components contribute to the cross section:

$$\frac{d\sigma_{ij}}{d\cos\theta} \approx \frac{d\sigma_{ij}^{LL}}{d\cos\theta} = \frac{\pi N_c N_f \alpha_W^2}{32 s}\left(\frac{1+\cos\theta}{2}\right)^2 \left(3 \mp e^{-2L_W(s)}\right) \tag{56}$$

Note that, for instance, $d\sigma^H_{\nu\bar{e}} > d\sigma^H_{e\bar{e}}$. Therefore, the BN violating corrections *increase* the physically relevant $e\bar{e}$ cross section, that reaches in the asymptotic limit of very high energy the isospin average. From (56) we also see that in the $g' \to 0$ limit, angular and energy dependences are factorized. Therefore in this limit the forward-backward asymmetry for $e\bar{e}$ is equal to the tree level value of $\frac{3}{4}$. However, the g'^4 terms are not completely negligible, producing a relative correction to A_{FB} of about 1.8 % at 1 TeV, the dependence on energy being given by the by now familiar universal behavior.

From (55) and taking into account the g'^4 terms also, we obtain the relative effect for for $e_L\bar{e}_L \to$ hadrons:

$$(\frac{\Delta\sigma_{e\bar{e}}}{\sigma^H_{e\bar{e}}})^L = (\frac{\sigma^H_{\nu\bar{e}} - \sigma^H_{e\bar{e}}}{\sigma^H_{e\bar{e}}})^L\ (\frac{1-e^{-2L_W(s)}}{2}) \approx 0.8 L_W(s) \tag{57}$$

The effect for the unpolarized cross section is slightly reduced (we give also the first order QCD corrections):

$$\left(\frac{\Delta\sigma}{\sigma}\right)^{EW}_{e\bar{e}} \simeq 0.58\, L_W(s) = 0.58\,\frac{\alpha_W}{4\pi}\log^2\frac{s}{M^2} \qquad \left(\frac{\Delta\sigma}{\sigma}\right)^{QCD}_{e\bar{e}} \simeq \frac{\alpha_S}{\pi} \tag{58}$$

That is, radiative corrections to $e^+e^- \to$ hadrons of *weak* origin are bigger, at the TeV scale, than *strong* QCD corrections (see Fig. 2)

6.2 t - channel scattering

Here we consider processes in which the Born term is a t-channel scattering diagram involving only weak interactions. This involves processes like lq and $l\bar{q}$ scattering where l is a lepton and q a quark. We also discuss briefly the case for $\nu_\mu e$ scattering. The hard cross section is described in this case by the amplitude

$$\mathcal{M}_{\beta_1\beta_2,\alpha_1\alpha_2} \sim g'^2 yY\delta_{\beta_1\alpha_1}\delta_{\beta_2\alpha_2} + g^2 t^a_{\beta_1\alpha_1} t^a_{\beta_2\alpha_2} \tag{59}$$

where y, α_1 (Y, α_2) refer to lepton (quark) indices. By squaring and by summing over final states we obtain the projections of the overlap matrix for this case:

$$C_1^L = \frac{1}{2}g^2(g'^2 y_L Y_L \mp g^2\frac{1}{4}) \qquad C_0^L = \frac{3}{16}g^4 + g'^4 y_L^2 Y_L^2 \tag{60a}$$

$$B_0^R = g'^4 y_R^2 Y_L^2 \qquad B_0^L = g'^4 y_L^2 Y_R^2 \tag{60b}$$

$$A_0^R = g'^4 y_R^2 Y_R^2 \tag{60c}$$

This time, differently from Eqns. (54-55), the convention is that the upper index is for the initial lepton chirality. So B_0^R indicates an initial state with a R lepton and a L quark, and so on. The upper (lower) sign refers to the lq $(\bar{l}q)$ case.

The energy and angular dependence of the resummed cross section can be found as before:

$$\frac{d\sigma_{ij}}{d\cos\theta} = \frac{N_c}{128\,\pi s}\frac{s^2}{t^2}\left[(A_0^R + C_0^L \pm C_1^L e^{-2L_W(s)})(1+\cos\theta)^2 + 4\,B_0^R + 4\,B_0^L\right] \tag{61}$$

We can neglect the terms $\propto g'^4$ in (60), keeping the terms $\propto g^2 g'^2$ that are suppressed only by a factor $\frac{g'^2}{g^2} = t_W^2$ and therefore not negligible in general, and thus obtaining:

$$\frac{d\sigma_{ij}}{d\cos\theta} = \alpha_W^2\,\frac{N_c\,\pi}{2\,s}\frac{(1+\cos\theta)^2}{(1-\cos\theta)^2}\left(\frac{3}{16} + (-1)^{i+j}(\mp\frac{1}{8} + \frac{1}{2}y_L Y_L t_W^2)e^{-2L_W(s)}\right) \tag{62}$$

where the - sign refers to the lq and the + sign to the $\bar{l}q$ case.

Although the BN violating corrections are regulated by the universal Eqns. (51), it should be clear from (52) that their relative effect is dependent on the magnitude of σ_{12}^H and σ_{11}^H. In particular, in the case of left polarized beams one has:

$$\frac{\Delta\sigma_{\bar{e}u}}{\sigma_{\bar{e}u}^H} \simeq 2.98\,L_W(s) \quad \frac{\Delta\sigma_{\bar{e}d}}{\sigma_{\bar{e}d}^H} \simeq -0.75\,L_W(s) \quad \frac{\Delta\sigma_{eu}}{\sigma_{eu}^H} \simeq -0.84\,L_W(s) \quad \frac{\Delta\sigma_{ed}}{\sigma_{ed}^H} \simeq 5.4\,L_W(s) \tag{63a}$$

$$\frac{\Delta\sigma_{e\bar{\nu}_\mu}}{\sigma_{e\bar{\nu}_\mu}^H} = \frac{\Delta\sigma_{\nu_\mu\bar{e}}}{\sigma_{\nu_\mu\bar{e}}^H} \simeq 10.6\;L_W(s) \tag{63b}$$

The magnitude of the effect in the electron-muon antineutrino scattering in (63b) is given by the $\propto g'^2 g^2$ terms in (62). In fact, the cross sections for $\bar{e}u$ and $\bar{e}\nu_\mu$ are equal in the $g' \to 0$ limit. However, the different hypercharges for ν_μ, u and the signs in formulas (60,61) conspire to produce a particularly small hard cross section in the $\bar{e}\nu_\mu$ case.

Considering unpolarized beams, it is interesting to discuss the dependence of the effect on the scattering angle. As we can see from Fig. 3, there is no effect for $\cos\theta = -1$, the reason being that the LL component does not contribute to the cross section in this limit. The effect then increases greatly with the angle. A cutoff on the maximum value of $\cos\theta$ is necessary, since we always work in the limit $1 - |\cos\theta| \gg M^2/s$.

The physical process proceeds via a proton; then, since the effect has opposite signs for u and d quarks in (63b), the overall effect gets diminished. For instance, taking the simplest picture of an hadron constituted only by valence quarks, we obtain for polarized left handed beams:

$$\frac{\Delta\sigma_{ep}}{\sigma_{ep}^H} \approx \frac{2\Delta\sigma_{eu} + \Delta\sigma_{ed}}{2\sigma_{eu} + \sigma_{ed}} \simeq -0.39\;L_W(s) \qquad \frac{\Delta\sigma_{\bar{e}p}}{\sigma_{\bar{e}p}^H} \approx 0.5\;L_W(s) \tag{64}$$

This means that for hadron beams the relative effect is about one-half that for lepton beams in Eq.(57).

6.3 $q\bar{q}$ scattering

In the $q\bar{q}$ case the overlap matrix O_H contains, besides s- channel and t - channel contributions, also interference terms in the identical quarks case, and is characterized by the additional presence of QCD contributions, reguated by the strong coupling constant $\alpha_S(s)$, where $s \gg M^2$ denotes the hard process scale.

In order to have a preliminary understanding, we neglect EW contributions to O_H, and we note that EW BN violating corrections are absent of course in the gluon-gluon scattering cases, but also in the gluon - quark case (see (45) and discussion thereafter). In the example of pure $q_L\bar{q}_L$ s-channel annihilation on the other hand, we expect the possibility of having big relative effects at the parton level, because of the hierarchy between σ_{12} and σ_{11}. For instance, $\sigma^H_{u\bar{d}}$ vanishes in the limit considered here, while $\sigma^H_{u\bar{u}}$ is of order α_S^2, so that we obtain (in the LL case):

$$\sigma_{u_L\bar{u}_L} = \sigma_{d_L\bar{d}_L} = \frac{1}{2}\sigma^H_{u_L\bar{u}_L}\left(1 + e^{-2L_W(s)}\right) \tag{65}$$

$$\sigma_{u_L\bar{d}_L} = \sigma_{d_L\bar{u}_L} = \frac{1}{2}\sigma^H_{u_L\bar{u}_L}\left(1 - e^{-2L_W(s)}\right) \tag{66}$$

so that the corrections to, say, the cross section for $u_L\bar{u}_L$ is of the order of the cross section itself. However, one should be cautious here because the observable effect with hadron beams is not really large. Considering for instance the case of valence quarks in $p\bar{p}$ collisions we first take into account that BN violating effects involve only left particles, so that the unpolarized parton level cross sections are:

$$\sigma_{u\bar{u}} = \sigma_{d\bar{d}} = \sigma_{u_L\bar{u}_L} + \sigma_{u_R\bar{u}_R} = \sigma^H_{u\bar{u}}\left(\frac{1 + e^{-2L_W(s)}}{4} + \frac{1}{2}\right) \tag{67a}$$

$$\sigma_{u\bar{d}} = \sigma_{d\bar{u}} = \sigma_{u_L\bar{d}_L} = \sigma^H_{u\bar{u}}\left(\frac{1 - e^{-2L_W(s)}}{4}\right) \tag{67b}$$

Next, a further suppression of the effect comes about because of a partial cancellation between $u\bar{u}$ and $u\bar{d}$ channels. In fact, since scattering occurs with probability $\frac{5}{9}$ in a $u\bar{u}$ or $d\bar{d}$ configuration and $\frac{4}{9}$ in a $u\bar{d}$ or $d\bar{u}$ configuration, we obtain:

$$\sigma_{p\bar{p}} \approx \frac{5}{9}\sigma_{u\bar{u}} + \frac{4}{9}\sigma_{u\bar{d}} = \sigma^H_{p\bar{p}}\left(1 + \frac{e^{-2L_W(s)} - 1}{20}\right) \quad ; \quad \frac{\Delta\sigma_{p\bar{p}}}{\sigma^H_{p\bar{p}}} \approx -\frac{1}{10}L_W(s) \tag{68}$$

and about twice as much in the pure LL channel. Therefore, the relative effect is reduced to the *one percent* range at the TeV threshold.

7 Outlook

The outcome of this paper is that electroweak corrections become pretty large at the TeV scale, even for inclusive processes, because of uncancelled double logarithmic enhancements involving the effective coupling $L_W(s)$. This leads to a sort of early unification within the Standard Model itself, because strong and EW corrections are to be considered together much before the respective running couplings become comparable.

From a theoretical point of view, the effect above is due to both the nonabelian nature of the $SU(2)$ component of the standard model and to symmetry breaking itself, which allows the initial states to be prepared as abelian charges. It follows that all initial states carrying nontrivial weak isospin are affected by the uncancelled double logs. For initial fermions we find the following features:

(i)Only left-handed doublets are affected. This means that the BN violating effect is strongly polarization dependent, and may lead to nontrivial angular dependence in some cases (Fig. 3).

(ii)The effect is particularly important for purely leptonic beams, for which its size is directly provided by $L_W(s)$, which is about 7 % at the TeV threshold. For instance, in the case of $e^+_L e^-_L \to$ hadrons, despite some

reduction by exponentiation and running coupling effects, EW corrections are already 5.2 %, compared to 3 % strong corrections (Fig. 2).

(iii)There is a composite state reduction of the effect for hadron beams, which act in the hard process as mixtures of partonic isospin states (recall that the effect vanishes for a mixture with equal weights). The suppression is of about 50 % in the $lp, \bar{l}p$ scattering processes considered in Sec. 6.2. It is even stronger for hadron-hadron beams, due to both initial isospin averaging and to QCD being flavor blind. Nevertheless, sizeable effects in the *percent* range are still expected in the $p\bar{p}$ case (Sec. 6.3). The more relevant pp case requires a detailed analysis, in which both structure functions and boson initiated processes will presumably play a role.

Undoubtedly, quantitative estimates of the effects presented here are needed for the planning of future accelerators. This implies not only a more detailed analysis of Born cross sections, but also an extension to subleading corrections which is far from being trivial [4, 5]. What we learn here is that even at inclusive level we need to revise the factorization theorems we are used to in QCD, in order to cope with fixed initial flavor and to disentangle the enhanced SM corrections from new physics.

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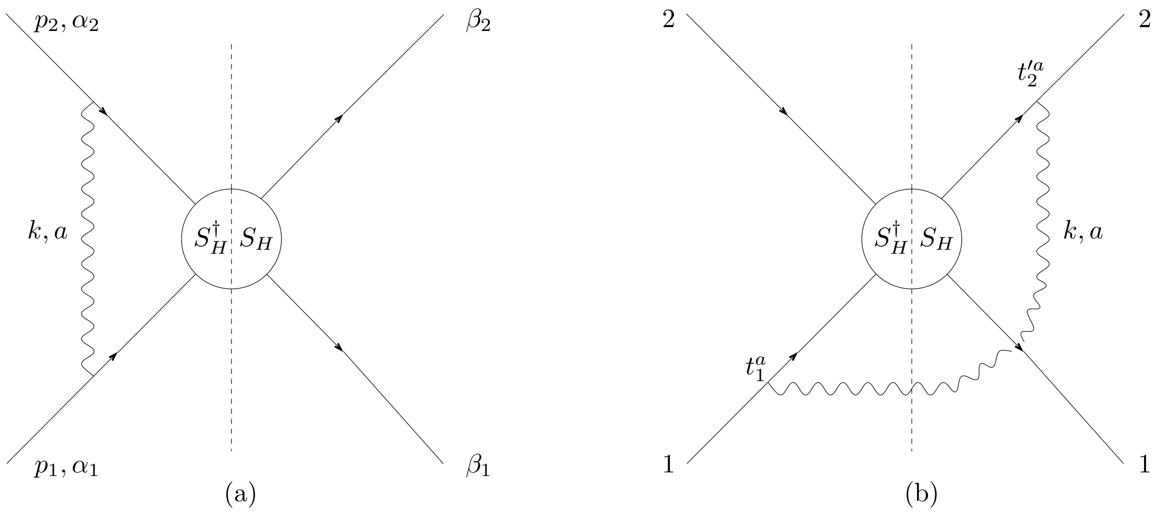

Figure 1: Unitarity diagrams for (a) virtual and (b) real emission contributions to lowest order initial state interactions in the Feynman gauge. Sum over gauge bosons a= γ, Z, W and over permutations is understood.

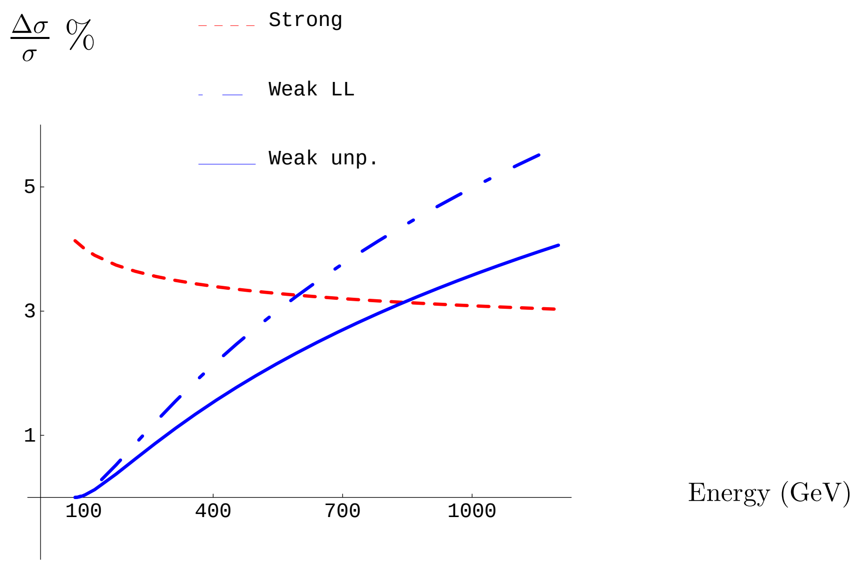

Figure 2: Resummed double log EW corrections to $e^+e^- \to$ hadrons and strong corrections (dashed line) up to 3 loops. The dash-dotted line is for a LL polarized beam, while the continuos line is for an unpolarized beam.

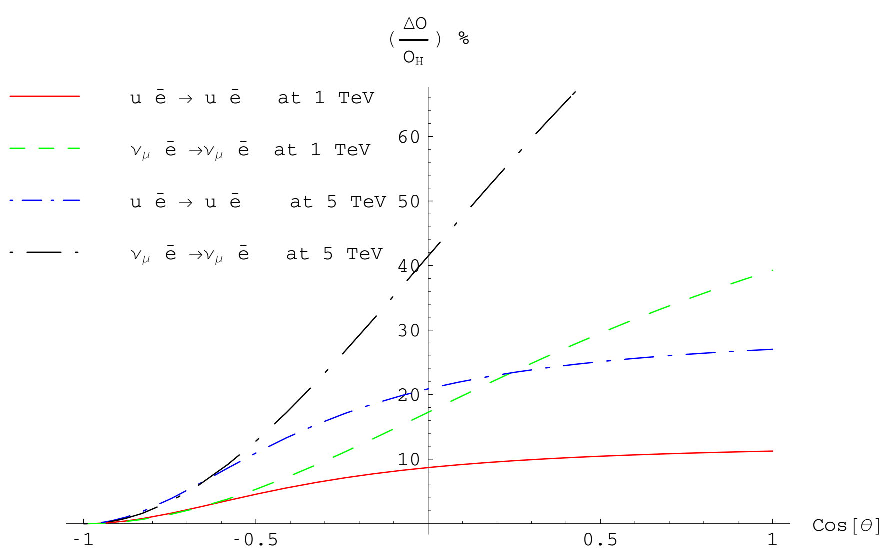

Figure 3: Relative effects for $O = \frac{d\sigma}{d\cos\theta}(\nu_\mu \bar{e})$ and $O = \frac{d\sigma}{d\cos\theta}(u\bar{e})$ at $\sqrt{s} = 1$ TeV and $\sqrt{s} = 5$ TeV

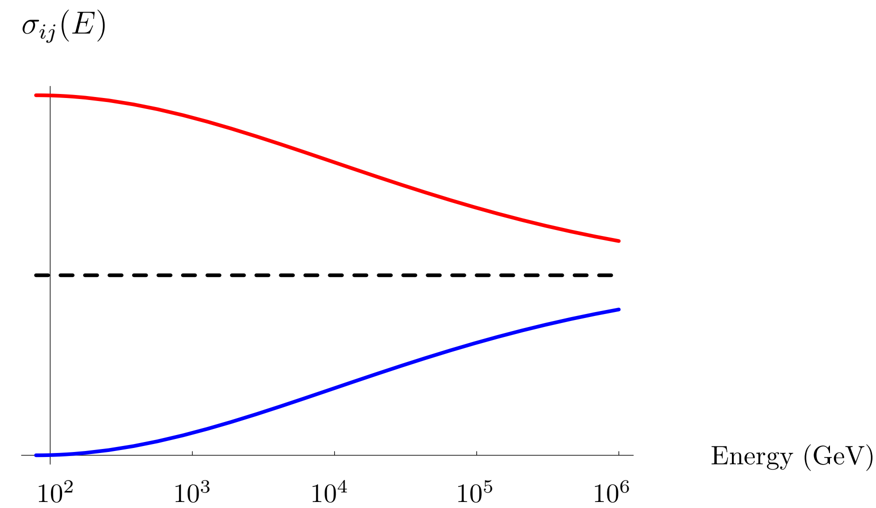

Figure 4: σ_{12} and σ_{11} as a function of energy. The vertical scale is arbitrary

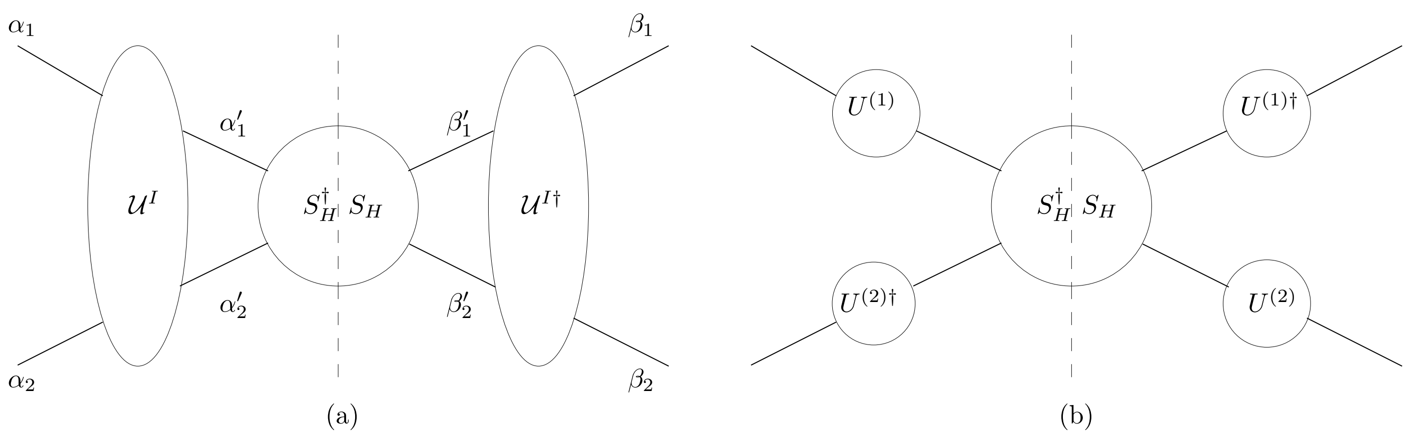

Figure 5: Soft dressing of the hard S-matrix S_H is described by the coherent state operator $\mathcal{U}^I$ (a). At leading order, the latter is factorized into leg operators $U^{(i)}$ (b).

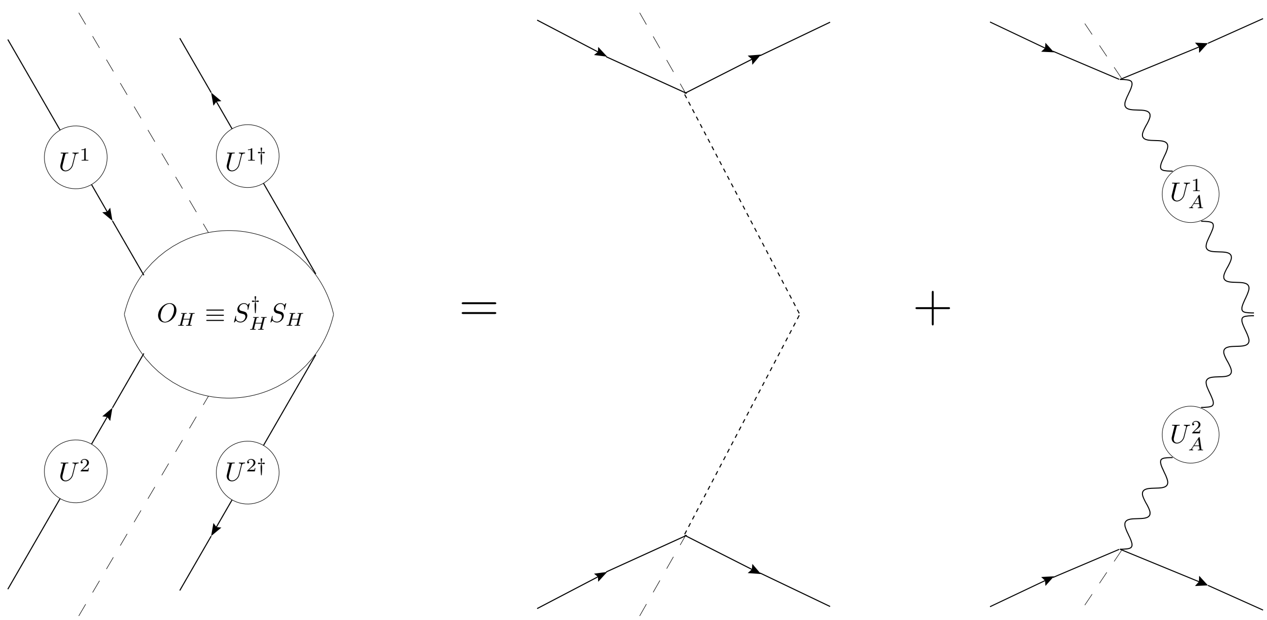

Figure 6: Singlet and vector decomposition of the overlap matrix O_H in the t-channel. The adjoint coherent state dressing is depicted also